PAPER • **OPEN ACCESS**

Model-Eliciting Activities: Engaging students to make sense of the world

To cite this article: L Pertamawati and E Retnowati 2019 *J. Phys.: Conf. Ser.* **1200** 012003

Model-Eliciting Activities: Engaging students to make sense of the world

L Pertamawati [1]**, E Retnowati** [2]

[1] Graduate School of Mathematics Education, Universitas Negeri Yogyakarta, Indonesia
[2] Department of Mathematics Education, Faculty of Mathematics and Natural Science, Universitas Negeri Yogyakarta, Indonesia

Email: [1] pertamawati.2017@student.uny.ac.id, [2] e.retno@uny.ac.id

Abstract. Model-Eliciting Activities (MEAs) demand students to construct, test and revise mathematical modelling during the learning process. In MEAs, students are provided open-ended questions using real life contexts. Six principles that are essential in the activity include Personal Meaningfulness, Model Construction, Self Evaluation, Model Documentation, Effective Prototype, as well as Model Share-ability and Reusability. This literature study describes briefly how MEAs are able to engage students developing connection between mathematics and the world. It is indicated that the contexts might assist students in making sense of mathematics and build relation with real life problems. An example is given to illustrate the Model-Eliciting Activities using a topic in Trigonometry.

1. Introduction

In a number of countries, mathematics is a compulsory subject at school. Therefore, students prefer to learn mathematics to pass examinations rather than to mastering mathematics material on its own. The consequence of this behaviour is worrisome as there are a great number of students who are not capable of applying what they have studied after they graduated from school [1–3]. The characteristic of real life problems, with workplace problems, in particular, are far different with problems faced in school mathematics [4,5]. It is stated that mathematics which is taught in school is not adequate enough to facilitate people to solve problems in workplace settings [1,3,6]. By the importance of skill to apply mathematical concepts in real life, it therefore becomes a necessity to provide students with learning experience encouraging complex real life problem solving.

This literature study aims to describe briefly how Model-Eliciting Activities (MEAs) act as alternative to promote sense making. Sense making is a crucial part to develop students' understanding of mathematical concepts. It also contributes to developing students' reasoning in solving mathematical problems. By the understanding mathematical concepts and the reasoning behind problem solving activities, it is hoped that students will be able to apply their mathematical knowledge, whether it is in the workplace setting, or in general, in the real world.

2. Sense Making in Mathematics Learning Practice

Sense making is linked to the ability to connect existing knowledge to understand the situation and concept in learning mathematics [7]. Being an integral aspect of reasoning ability, sense making is a crucial part to encourage students in solving mathematical problems. Students learn best when they are engaged with the contexts of the problems. Therefore, the implementation of teaching instructions which weight on developing students sense making in learning mathematics should be promoted and encouraged in order to achieve maximum learning outcomes.

Several studies have stated that sense making is a dynamic process which affects individual preferred action in the real world [7–11]. Sense making needs to be introduced to students from an early age, not only because it motivates students to learn mathematics further, but it also helps students to organize and understand mathematical concepts and its relation to the real world.

Specifically, sense making serves as a tool to organize one's mind, including noticing and bracketing information, labelling information, getting retrospective, making presumption, deciding action, while trying to communicate the understanding to the world [10].

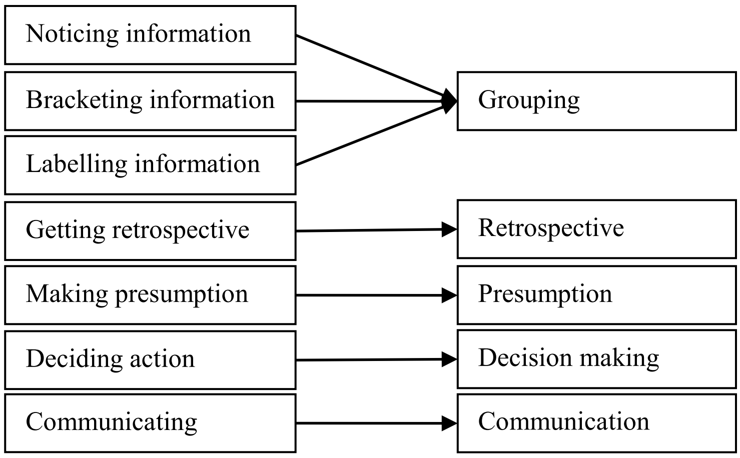

Figure 1. Sense Making Keywords

Sense making is involved in all steps of the thinking process. Summarizing the components of sense making, there are five keywords that demonstrate the existence of sense making in learning mathematics. These five keywords are grouping, retrospective, presumption, decision making, and communication. The descriptions of the keywords are delivered below.

Table 1. The Descriptions of Sense-Making Keywords

Keywords	Descriptions
Grouping	The ability to recognize, classify and identify mathematical information.
Retrospective	The ability to connect mathematical knowledge and experiences with the new mathematical information
Presumption	The ability to believe that the mathematical information which is processed is true and valid.
Decision making	The ability to integrate information which is provided in the learning process to decide further steps of problem solving activity.
Communication	The ability to deliver thought process effectively whether it is in oral or written form.

3. Model-Eliciting Activities as Case Studies for Students

Researchers in mathematics education and practice have promoted a number of learning activities to introduce real life problem solving strategies. The activities which are going to be talked about further are Model-Eliciting Activities (MEAs). MEAs first came into the surface by the work of researcher, teacher and education enthusiasts which focuses on the mathematical modelling construction by the students [12]. MEAs are chosen as suitable activities to promote real life problem solving as mathematical modelling serves as a connection between real life and mathematical concepts [13–15].

In addition, mathematical modelling in education has become a great concern in the past decades. However, there is still a small number of learning practices which focuses on the construction of mathematical modelling by the students. It is unfortunate because international assessments, including PISA and TIMSS, indicate mathematical skills by assessing mathematical modelling competency of the participants.

As an open-ended task, the model constructed in mathematical modelling is a product of students' individual strategies and documentation of thought process during problem solving activity [16–20]. Differ from word problems, modelling is a dynamic process. It encourages students to test and continuously revise the model until the best model is formed. This characteristic of mathematical modelling is what makes teachers hesitate to apply learning methods which explicitly involve modelling activity. The original modelling activity is complex and time-consuming which is why MEAs are suggested. MEAs are simplified modelling tasks applied to students and kids in general. While still encouraging students to construct a mathematical model in learning mathematics, MEAs are briefer in the context and demanding less time to be implemented.

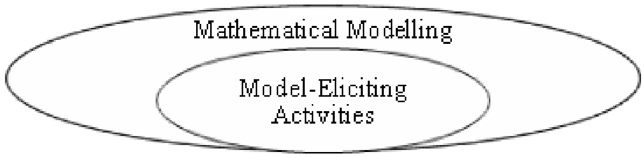

Figure 2. Model-Eliciting Activities (MEAs) and Mathematical Modelling

MEAs weight on students' participation in constructing, testing, and revising mathematical model by engaging real life contexts [20]. Called as case studies for students, students are given an opportunity to construct their own strategies [21]. Therefore, MEAs are very suitable to promote sense making and encouraging students' constructive process.

MEAs utilize real problems which are stimulated from newspaper articles or other media that are more real and more understandable to students in terms of concrete facts and media presentations. The activities encourage students to analyze and research the problems by their own existing knowledge. Students are expected to explore the problems and discover mathematical concepts during MEAs. To support the activities, there are six principles to ensure MEAs are successful in assisting students to learn mathematics [12]. The six principles are briefly explained below [22].

3.1. Personal Meaningfulness
The use of relatable and meaningful lifelike problems is encouraged by this principle. Students need to understand and imagine the problems which are given by the teacher. It is important to choose the right problems in order to make students relate to the problems well.

3.2. Model Construction
This principle reminds teacher not to choose problems which can be solved directly using an intended formula. This principle demands students to realize that there is a need to construct a new way of solving the problem. Students need to find new patterns and structures from the problem.

3.3. Self Evaluation
This principle encourages students to evaluate their problem solving strategies. It is important to make sure that the problems must be able to encourage students in assessing the purpose of solving the problem. Students need to recognize the accuracy of the solution by their problem solving activities.

3.4. Model Documentation
This principle demands students to document their problem solving strategies. Teachers have to ensure that the problems are able to make students explain their thoughts in detail, whether it is in oral or in written form.

3.5. Effective Prototype
This principle emphasizes that students are able to create effective models that can solve the problem. The problem is expected to be a relatively simple problem, but it is mathematically meaningful and is able to make students think and produce effective models.

3.6. Model Share-ability and Reusability
This principle ensures that the problems are relevant to other similar problems. By the end of the learning activity, it can help students to solve similar problems with minimum efforts. The solution models which are obtained are re-usable, able to get modified and able to be applied to analogue situations. Furthermore, students are expected to be able to share what they get. At the end of the problem solving activities, students are asked to present and exchange their solution model to their peers.

The six principles of MEAs are essential to encourage students in developing their sense making of mathematical concepts. In addition to these six principles, there are four essential steps that differ model-eliciting activities from traditional word problems used in everyday teaching and learning mathematics in school. The essential steps which are capable to develop students' sense making in mathematics learning include newspaper articles, warm-up questions, mathematical information, and problem solving task [21]. A brief explanation of these essential parts is given in the next section.

4. Promoting Sense Making by MEAs
Modelling acts as a tool to integrate mathematical concepts with the real world, therefore, it focusses on sense making of the students. MEAs do not only demand students to understand mathematical concepts and choose the right procedure in solving the problem, but the activities also encourage students to transform real life situation into a mathematical situation. Students need to solve a mathematical problem in order to be able to transfer mathematical solution back to reality so the solution makes sense for them and other people. The implementation of MEAs is characterized by the use of newspaper article, warm-up questions, mathematical information and problem solving task given by the teacher [21].

Newspaper article serves as a bridge to students in understanding the topic that is going to be learnt further. It gives an illustration to students about the problem in real life. The next step is characterized by the assistance of teacher as students answer some warm-up questions regarding the article. The teacher should make sure that the information in the article makes sense after students answer the entire warm-up questions. Next step of the activity is attributed to mathematics information which is delivered to the students. Students are given data, tables, and relevant mathematical information. The mathematical information helps students to recognize patterns and structures to solve the problems further. The next procedure is the most important step that is delivering problem solving task. Problems are delivered as open-ended questions to encourage diverse solutions and reasonings by the students. It is important to notice that there is not always a single solution and reasoning to the problems.

Furthermore, based on the brief description of the principles of MEAs above, it can be noted that MEAs are learning activities which are in line with the development of sense making by the students. Below are the principles of MEAs categorized with sense making keywords.

IOP Conf. Series: Journal of Physics: Conf. Series **1200** (2019) 012003 doi:10.1088/1742-6596/1200/1/012003

Table 2. MEAs and Sense Making

Principles	Descriptions of Sense Making Keywords in Learning Process
Personal Meaningfulness	Students investigate the truth of the situation in real life based on their experiences. (*presumption*)
Model Construction	Students describe, explain, manipulate, predict, and control a contextual system that is relevant to the problem (*grouping*)
Self Evaluation	Students assess whether the solution is adequate enough to solve the problem. If the solution is not able to solve the problems, students can revise, manipulate and improve the model solution. (*retrospective*)
Model Documentation	Students document and explain the mathematical model which is obtained so that it is understood by other people. (*communication*)
Effective Prototype	Students create a model as a solution that can be used again on problems with the same characteristics. (*decision making*)
Model Share-ability and Reusability	Students communicate the mathematical model which is obtained and make sure that the solution is applicable to similar problems. (*presumption)*

5. Example of MEAs

5.1. Design of the activity: Submarine SONAR Sound

The example below is inspired by submarine sonar sound which is hopefully can be imagined by students.

5.1.1. Newspaper article. Students are demanded to observe the article to give a brief introduction to the upcoming task that is going to be solved. The article is shown in Figure 3.

How Submarines Work

Have you ever imagined how submarines navigate through the sea? Submarines are equipped with navigational charts and equipments. To navigate on the surface, Global Positioning System (GPS) may determine latitude and longitude, but this equipment may not work underwater. To locate a target underwater, submarines use active and passive sound navigation and ranging (SONAR).

Passive SONAR consists of an underwater microphone which is dropped into the water to listen to the sound of the target. The problem of passive SONAR is that it only works when the target moves. It does not work when the target is silent and does not make any movement. Recognizing this problem, the use of active SONAR was then introduced. Active SONAR uses its own sound waves to navigate target underwater. When the sound of active SONAR strikes the target, the sound waves are reflected back to return to the ship. The active SONAR equipment records the speed of sound in water and the time for the sound waves to return to the ship. By knowing the speed of sound in water and the time for the sound waves to return to the ship, the distance between the submarines and the target can be calculated

Figure 3. An article serves to introduce the content to the students

5.1.2. Warm-up questions. After receiving sufficient information about how submarines work, students are being asked about several basic questions regarding SONAR, which is used to measure the distance of objects under the sea. The questions are shown in Figure 4.

IOP Conf. Series: Journal of Physics: Conf. Series **1200** (2019) 012003 doi:10.1088/1742-6596/1200/1/012003

Based on the article, could you explain:
1. How submarines navigate the target through the deep sea?
2. How does SONAR work?
3. What are the characteristics of SONAR?

Figure 4. Warm-up questions to confirm students' understanding of the topic

5.1.3. Mathematical information. Mathematical information is delivered to give an opportunity for students to generate and discover patterns that will lead to a mathematical concept. The example of mathematical information about submarines is given in Figure 5 below.

Below are the data gathered on submarines by the use of SONAR in the past 2 months.

Date	Time	Distance (m)	Depth (m)	Elevation (^{0})*
01/02/2018	10:12:25	1296	800	45
01/09/2018	15:12:25	160	500	60
01/16/2018	09:12:25	199	420	35
01/22/2018	08:12:25	810	500	45
01/29/2018	20:05:30	130	405	60
02/05/2018	17:23:25	166	350	35
02/12/2018	14:27:18	982	606	45
02/19/2018	13:10:40	261	550	35
02/25/2018	19:27:30	224	700	60
01/02/2018	10:12:25	1296	800	45

* the elevation is the angle between sonar slant range and the sea level.

Figure 5. Mathematical information which is related to the topic

5.1.4. Problem solving task. Students are given an opportunity to discuss and solve the problem solving task within groups consists of 3-4 people. The teacher is encouraged to support students' independent learning process. Direct instructions and interventions are not suggested in order to develop students' problem solving skills. Figure 6 shows problem solving task in the topic.

Based on the mathematical information which has been given earlier, can you explain:
1. How can we predict the distances of the objects located under the sea?
2. Is there any characteristic differing each of the distances which are calculated by SONAR?
3. How do you decide whether objects underneath the sea level are observable by submarine's SONAR? What is possible and what is not possible?

Figure 6. Problem solving task which is related to the topic

5.2. How students make sense of the task

It is indicated that the problem solving activity requires sense making in order to solve the problem realistically. The descriptions of the sense making that may happen to the students are delivered below.

*5.2.1. Grouping***.** Students classify mathematical information based on the similarity and difference. Not only students are able to relate the problem to real life situation, but students are also encouraged to find patterns and structures from mathematical information provided by the problem. Recognizing the importance of distance, depths and elevations in solving problems added by the finding of similarities and differences between these three components would prepare students to proceed to the further step of solving the problem.

5.2.2. Retrospective. Students are encouraged to evaluate their problem solving strategies and solutions. It is highly suggested for the teacher to facilitate students in reconstructing and revising the way students come to the best solution of the problem. Students need to be able to generalize the mathematical relationship between each distance, depths and elevations on the active sonar data which are given in the task. It is assumed that in this stage of learning, students are able to utilize the understanding about the ratio of triangle to predict how they are going to solve the problem.

5.2.3. Presumption. Students assume that the solution is able to solve or predict another analogue real life problem in the future. It is important to convince the students to believe that they are able to utilize the ratios between distances, depths and elevations in order to predict future occasion in different contexts.

5.2.4. Decision making. Students decide the most suitable problem solving strategy and continuously revise it in order to find the best solution to the problem. Students are able to analyze whether the problem solving steps and the solution obtained is valid.

5.2.5. Communication. Students explain their problem solving strategies in the form of group reports and present it in front of the class. It is important to let students observe and exchange diverse strategies between different groups. At the end of the discussion, it is a necessity for the teacher to explicitly introduce the context of trigonometry, including its notations and symbols.

6. Conclusion

Model-Eliciting Activities (MEAs) are learning activities which consist of six principles. The activities act as alternative to constructive learning in mathematics. An MEA naturally engages students by its relevant and meaningful principles. Particularly acts as problem solving activity, it serves as a tool to develop sense making as it supports students to independently identify mathematical information, evaluate problem solving strategies, make generalization based on the contexts, develop decision making skill, and communicate thought process in learning mathematics. This literature study suggests that the implementation of MEAs is able to prepare students in solving real life problems in the future by applying mathematical concepts which have been studied in school. Further study to implement MEAs in mathematics teaching and learning practice is highly encouraged to support the finding of this study.

References

[1] Gravemeijer K, Stephan M, Julie C, Lin F, and Ohtani M 2017 *Int. J. Sci. Math. Educ.* **15** 105–23.
[2] FitzSimons G 2013 *Adult Learning Math.: An Int. J.* **8** 7–19.
[3] Wedege T 2002 *For the Learning of Math.* **22** 23–8.
[4] Lave J 1988 *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life* (Cambridge: Cambridge University Press)
[5] Nunes T, Schliemann T and Carraher D 1993 *Street Mathematics and School Mathematics* (Cambridge: Cambridge University Press)
[6] Mills K 2012 *Proc. of the 19th Int. Conf. of Adult Learning Mathematics - A Research Forum (Auckland, New Zealand, 26-29 June 2012)* ed A Hector-Mason and D Coben (Auckland: The National Centre of Literacy and Numeracy for Adults, University of Waikato) pp 69–83
[7] National Council of Teachers of Mathematics 2009 *A Teacher's Guide to Reasoning and Sense Making* (Reston, VA: NCTM)
[8] Coburn C E 2001 *Educ. Eval. Policy Anal.* **23** 145–70

[9] März V and Kelchtermans G 2013 *Teaching and Teacher Educ.* **29** 13–24
[10] Spillane J P, Reiser B J and Reimer T 2002 *Rev. Educ. Res.* **72** 387–431
[11] Weick K E, Sutcliffe K M and Obstfeld D 2005 *Organ. Sci.* **16** 409–21
[12] Lesh R, Hoover M, Hole B, Kelly A and Post T 2000 *Handbook of Research Design in Mathematics and Science Education* ed A Kelly and R Lesh (Mahwah, NJ: Lawrence Erlbaum) pp 591–646
[13] Asempapa R S 2015 *J. of Math. Educ.* **8** 16–29
[14] Blomhøj M 2004 *Int. Perspectives on Learning and Teaching Mathematics* ed B Clarke *et al* (Göteborg University, Sweden: National Center for Mathematics Education) pp 145–60
[15] Chan E C M 2008 *The Math. Educator* **11** 47–66
[16] Blum W and Borromeo-Ferri R 2009 *J. of Mathematical Modeling and Application* **1** 45–58
[17] Chamberlin S A and Moon S M 2008 *Int. J. of Math. Teaching and Learning* **9** 78–105
[18] Lesh R and Harel G 2003 *Math. Thinking and Learning: An Int. J.* **5** 157–190
[19] Lesh R A and Zawojewski J S 2007 *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* ed F Lester (Charlotte, NC: Information Age Publishing) pp 763–804
[20] Lesh R and Doerr H M 2003 *Beyond Constructivism: Models and Modelling Perspectives on Mathematics Problem Solving, Learning, and Teaching* ed R Lesh and H M Doerr (Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.) pp 3–33
[21] Chamberlin S A and Moon S M 2005 *J. of Second. Gift. Educ.* **17** 37–47
[22] Chamberlin S A and Coxbill E 2012 *Quantitative Reasoning and Mathematical Modeling: A Driver for STEM Integrated Education and Teaching in Context* (*WISDOMe Monograph Series* vol 2) ed L Hatfield and R Mayes (Wyoming: Wyoming Insititute for the Study of Mathematics Education) pp 169–79